Dynamical Behaviors and Chaos Control of a Non-Linear System

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Abstract

Some basic dynamical properties of a three-dimensional nonlinear chaotic system are analyzed by means of numerical simulations and theoretical analysis. Furthermore, the nonlinear dynamical behaviors of the nonlinear system are studied via phase diagram, Poincare mapping, and power spectrum. Finally, a controller of this system is constructed, and the forming mechanism of the chaotic attractor is demonstrated by Lyapunov exponents and bifurcation diagram. Our discussion shows that the chaotic attractors of the Lorenz-like system can be controlled well by changing the control parameter. This chaotic attractor is a compound structure obtained by merging together two simple attractors after a mirror operation.

Keywords

Chaotic attractor; Forming mechanism; Lyapunov exponent; Bifurcation diagram.

1. Introduction

In 1963, Lorenz found the first chaotic attractor in a three-dimensional (3D) autonomous system when he studied atmospheric convection [1]. From that time, different chaotic systems continuously were found [2–9], and furthermore the bifurcation, chaotic behaviors, structure of chaotic attractor and procreant condition of chaos are systemically studied [10–17]. Chen and Ueta constructed a 3D autonomous chaotic system from an engineering feedback control approach in 1999 [2]. Lu et al found a new chaotic system soon in 2002 [3], a unified system that combines the three systems as its special cases [18]. According to the canonical-form classification defined by Vanecek and Celikovsky [19], the linear part $A=[a_{ij}]_{3\times3}$ of 3D autonomous systems with quadratic nonlinearities shows distinct features for different systems, $a_{12}a_{21}>0$ for the lorenz system, and $a_{12}a_{21}<0$ for the Chen system, while the Lu system satisfies $a_{12}a_{21}=0$. Thus the three systems are not topologically equivalent. In this sense, they together constitute a complete family of the generalized Lorenz dynamical systems.

Based on the previous analysis, a new 3D continuous autonomous Lorenz-like chaotic system is constructed in this paper. The nonlinear dynamical method is used to study the dynamical behaviors of this system, some basic dynamical properties of the system are studied by means of numerical simulations and theoretical analysis. Furthermore, the chaotic behaviors of the nonlinear system is numerically simulated via phase diagram, Poincare mapping and power spectrum. Finally, a controller of this system is constructed, and the forming mechanism of chaotic attractor is studied. Moreover, the chaotic attractor of the Lorenz-like system can be controlled well by changing the control parameter. This new chaotic attractor is a compound structure obtained by merging together two simple attractors after a mirror operation.

2. Model

The famous Lorenz system [1] is

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$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - y - xz, \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

Which has a chaotic attractor when parameters a=10, b=8/3, c=28.

Here we propose a new three-dimensional chaotic system, of which the autonomy differential equations is described by

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = c(x + y) - xz, \\ \dot{z} = -bz + x^2, \end{cases} \tag{2}$$

here, x, y, z are the state variables of the system (2), and a, b, c are real constant parameters. Let a=28.0, b=2.5, c=19.0, the system (2) has chaotic attractor as shown in Figure 1(a).

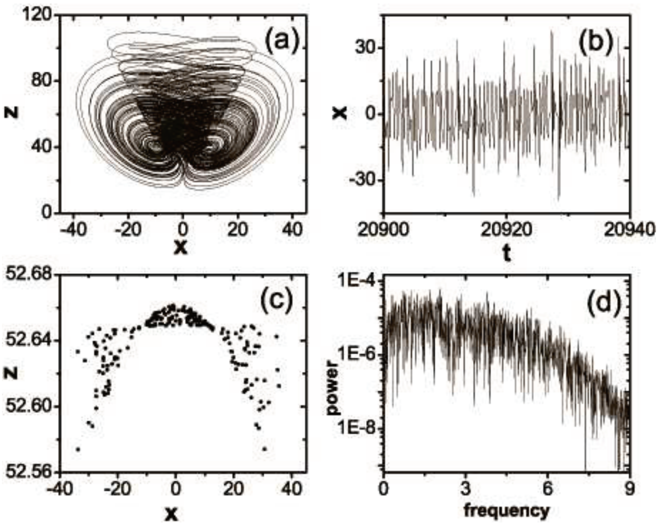

Figure 1. (color online) (a) The typical strange attractor of the system (2) for x−z phase plan. (b) The time-series diagram of the system (2) for x(t). (c) The Poincare mapping of the system (2) for x−z plane. (d) The power spectrum of the system (2) for |x|. The parameters of the system (2): a=28.0, b=2.5, and c=19.0.

3. Basic Dynamical Properties

The basic dynamical properties of the system (2) will be analyzed explicitly in this section.

3.1. Symmetry of the System (2)

The symmetry property widely exists in this dynamic system. We find that the system (2) has a natural symmetry, namely doing transform (x, y, z) → (−x, −y, z), that is to say, the system (2) is symmetrical anent z axis. Moreover, the symmetry is tenable for all parameters.

3.2. Stability of the system (2)

The stability of the system (2) is analysed by Lyapunov method. First, we discuss equilibria of this nonlinear system (2). Let

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$$\begin{cases} a(y-x)=0, \\ c(x+y)-xz=0, \\ x^2-bz=0, \end{cases} \tag{3}$$

Obviously, the equations (3) is a nonlinear algebraic equations, the system has three equilibria, which are respectively described as follows $O(0,\ 0,\ 0)$, $P^+(x_0,\ y_0,\ z_0)$, $P^-(x_1,\ y_1,\ z_1)$. We operate above these nonlinear algebraic equations and obtain $O(0,\ 0,\ 0)$, $P^+(9.7468,\ 9.7468,\ 38)$, $P^-(-9.7468,\ -9.7468,\ 38)$. For equilibrium point $O(0,\ 0,\ 0)$, system (2) are linearized, and the Jacobian matrix is defined as

$$J_0=\begin{bmatrix} -a & a & 0 \\ c-z & c & -x \\ 2x & 0 & -b \end{bmatrix}=\begin{bmatrix} -28 & 28 & 0 \\ 19 & 19 & 0 \\ 0 & 0 & -2.5 \end{bmatrix}. \tag{4}$$

To gain its eigenvalues, let $|\lambda I-J_0|=0$. These eigenvalues that corresponding to equilibrium $O(0,\ 0,\ 0)$ are respectively obtained as follows $\lambda_1=-2.50,\ \lambda_2=-37.4280,\ \lambda_3=28.4280.$ Here λ_3 is a positive real number, λ_1 and λ_2 are two negative real numbers, therefore the equilibrium O(0, 0, 0) is a saddle point, so this equilibrium $O(0,\ 0,\ 0)$ is unstable.

Next, linearizing the system (2) about the other equilibria such as P^+ and P^- yields the following characteristic operation. For equilibrium point P^+, the Jacobian matrix equals to

$$J_+=\begin{bmatrix} -a & a & 0 \\ c-z & c & -x \\ 2x & 0 & -b \end{bmatrix}=\begin{bmatrix} -28 & 28 & 0 \\ -19 & 19 & -9.7468 \\ 0 & 0 & -2.5 \end{bmatrix}. \tag{5}$$

Let $|\lambda I-J_+|=0$, these corresponding eigenvalues of equilibrium P^+ are respectively obtained as follows $\lambda_1=14.0075,\ \lambda_2=-12.7538+14.7356i,\ \lambda_3=-12.7538-14.7356i.$ Here λ_1 is a positive real numbers, λ_2 and λ_3 become a pair of complex conjugate eigenvalues with negative real parts. Therefore the equilibrium P^+ is a saddle-focus, and so this equilibrium point is also unstable.

Similarly for equilibrium point P^-, the corresponding Jacobian matrix is

$$J_-=\begin{bmatrix} -a & a & 0 \\ c-z & c & -x \\ 2x & 0 & -b \end{bmatrix}=\begin{bmatrix} -28 & 28 & 0 \\ -19 & 19 & 9.7468 \\ -19.4936 & 0 & -2.5 \end{bmatrix}. \tag{6}$$

Let $|\lambda I-J_-|=0$, these corresponding eigenvalues of equilibrium P^- are $\lambda_1=14.0075,\ \lambda_2=-12.7538+14.7356i,\ \lambda_3=-12.7538-14.7356i.$ It is obvious that λ_1 is a positive real numbers, λ_2 and λ_3 are a pair of complex conjugate eigenvalues with negative real parts. Therefore the equilibrium P^- is also a saddle focus, so this equilibrium point is also unstable.

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The above brief analyses show that the three equilibria of the Lorenz-like chaotic system are all saddle focus nodes.

3.3. Dissipation of the System (2)

For the dynamical system (2), we can obtain

$$\Delta V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a - b + c = p, \tag{7}$$

with p=−a−b+c=−11.5, here p is a negative value. Dynamical system described by (2) is one dissipative system, and an exponential contraction of the system (2) is

$$\frac{dV}{dt} = e^{p} = e^{-11.5}. \tag{8}$$

In dynamical system (2), a volume element V_0 is apparently contracted by the flow into a volume element $V_0 e^{pt} = V_0 e^{-11.5t}$ in time t. It means that each volume containing the trajectory of this dynamical system shrinks to zero as $t \to \infty$ at an exponential rate p. So, all this dynamical system orbits are eventually confined to a specific subset that have zero volume, the asymptotic motion settles onto an attractor of the system (2), that means that the dynamical system (2) exists an attractor authentically.

4. Typical Chaotic Attractor

The chaotic behavior of system (2) is numerically simulated by means of a 4th-order Runge-Kutta algorithm with a time step δt=0.01 and the quantities of interest are measured after a sufficient long transient is discarded. Throughout the work, we let a=28.0, b=2.5 and c=19.0 unless specified. Initially, we assign the initial values of state variables as (2.2, 2.4, 28).

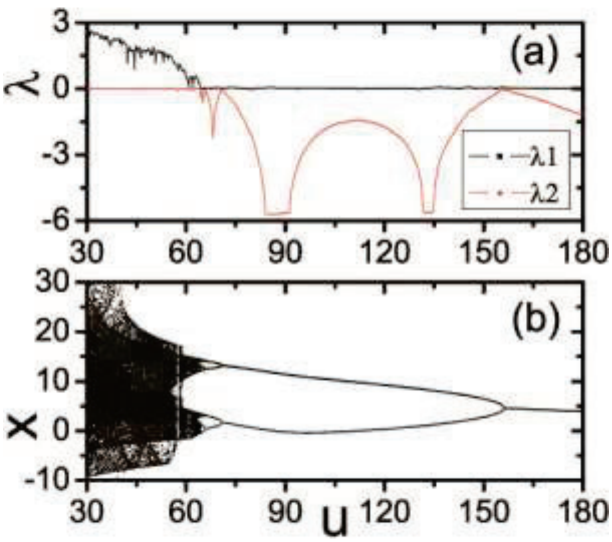

Figure 2. (color online) (a) The Lyapunov exponent spectrum of the system (2) (the black data is for the maximal Lyapunov exponent and the red data is for the second Lyapunov exponent). (b) The bifurcation diagram of the system (2). The control variable u is increase gradually. The parameters of the system (2): a=28.0, b=2.5, and c=19.0.

This nonlinear system exhibits complex and abundant dynamical behavior, as the phase portrait of the strange attractor is shown in Figure 1(a), which is similar to Lorenz chaotic attractors, but it is different to Lorenz system in the topological structure. The time-series diagram of x(t), Poincare mapping and power spectrum of the system (2) with parameters a=28.0, b=2.5, c=19.0 are presented in Figure 1(b), 1(c) and 1(d), respectively. The waveforms of x(t) in time domain obviously are non-periodic. Poincare mapping consists of some dispersed points as shown in Figure1(c). The power spectrum is continuous. From the above investigations, we know that the dynamics of the system (2) are chaotic.

5. Typical Chaotic Attractor Forming Mechanism Of The Lorenz-Like Chaotic Attractor

In order to reveal the forming mechanism of this new chaotic attractor structure, the controlled system is proposed. The differential equations of controlled system are expressed as

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = c(x+y) - xz + u, \\ \dot{z} = -bz + x^2. \end{cases} \tag{9}$$

In this system, u is the control parameter, of which the value can be changed within a certain range. With the variance of the parameter u, the chaotic behavior of this system (2) can effectively be controlled. Here, we let still initial values of the system (2) are selected as (2.2, 2.4, 28).

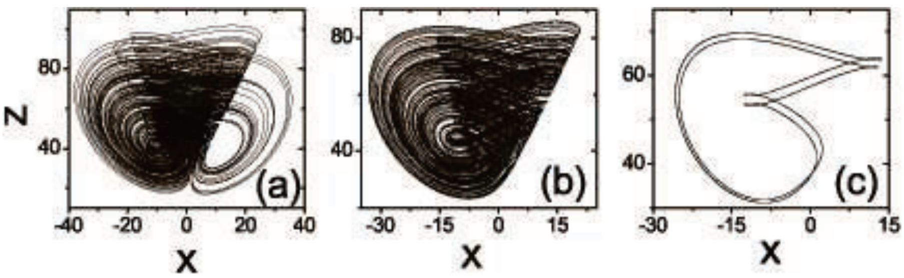

Figure 3. (color online) (a) x−z plane phase portrait of the system (2) with parameters a=28.0, b=2.5, c=19.0 and u=30.0. (b) x−z plane phase portrait of the system (2) with parameters a=28.0, b=2.5, c=19.0 and u=45.0. (c) x−z plane phase portrait of the system (2) with parameters a=28.0, b=2.5, c=19.0 and u=70.0.

5.1. Situation with Control Parameter $u \in [30,\ 180]$

First, we investigate the dynamics of the Eq(9) with control parameter $u \in [30,\ 180]$. For this aim, we consider the variation of the Lyapunov exponent spectrum and bifurcation diagram of the Eq(9). The results are presented in Figure.2 (a) and (b). With the increase of control parameter u, the maximal Lyapunov exponent spectrum (the black data is for the maximal Lyapunov exponent and the red data is for the second Lyapunov exponent in Figure2 (a) from greater than zero decreases to zero. A positive maximal Lyapunov exponent spectrum is usually taken as an indication that the system is chaotic. We can see clearly that the dynamics of the nonlinear system is from chaotic to periodic by period-doubling bifurcation in Figure2 (b) with the increase of control parameter u.

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To get more insight on the dynamics of the nonlinear system in different regimes and explore what happens when the system changes from one regime to another one. We calculate the phase trajectory for different regimes, respectively.

For u=30.0, the corresponding strange attractor is shown in Figure3 (a), which evolves into partial but is still bounded in this time.

For u=45.0, the corresponding strange attractor is shown in Figure3 (b). The attractor is evolved into single left scroll attractor, which is only one half the original chaotic attractor in this time.

For u=70.0, the strange attractor evolves into the period-doubling bifurcations, period-doubling phase portrait are shown in Figure 3 (c).

5.2. Situation with Control Parameter $u \in [-180,\ -30]$

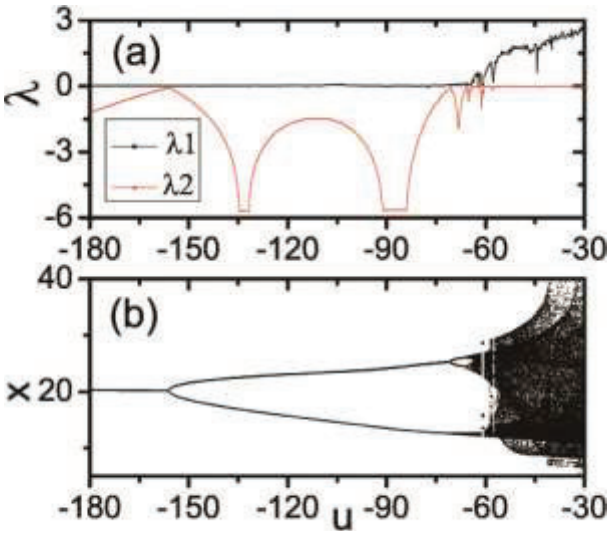

Figure 4. (color online) (a) The Lyapunov exponent spectrum of the system (2) (the black data is for the maximal Lyapunov exponent and the red data is for the second Lyapunov exponent). (b) The bifurcation diagram of the system (2). The control variable u is decrease gradually. The parameters of the system (2): a=28.0, b=2.5, and c=19.0.

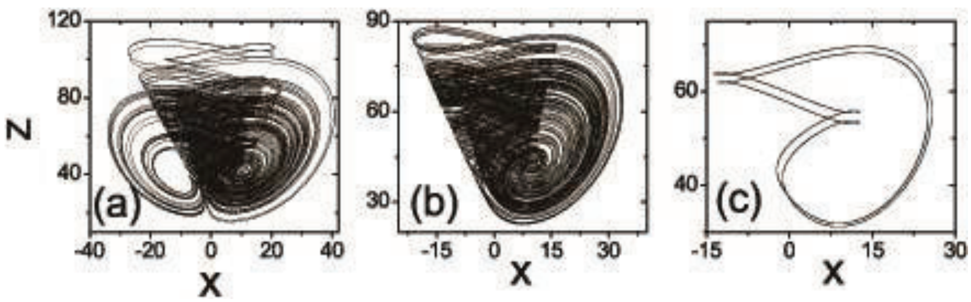

Figure 5. (color online) (a) x−z plane phase portrait of the system (2) with parameters a=28.0, b=2.5, c=19.0 and u=−30.0. (b) x−z plane phase portrait of the system (2) with parameters a=28.0, b=2.5, c=19.0 and u=−45.0. (c) x−z plane phase portrait of the system (2) with parameters a=28.0, b=2.5, c=19.0 and u=−70.0.

Now, the control parameter u is considered as a negative value. The variation of the Lyapunov exponent spectrum and bifurcation diagram of the Eq(9) are shown in Figure 4 (a) and (b). Correspondingly, we calculate the phase trajectory for different regimes, respectively.

For u=−30.0, the strange attractor is shown in Figure 5 (a), which evolves into partial but is still bounded in this time.

For u=−45.0, the corresponding strange attractor is shown in Figure 5 (b). The attractor is evolved into single right scroll attractor, which is only one half the original chaotic attractor in this time.

For u=−70.0, the strange attractor evolves into the period-doubling bifurcations, period-doubling phase portrait are shown in Figure 5 (c).

In the controller, chaotic attractor disappears when |u| is large enough; when |u| is small enough, a complete chaotic attractor appears. So |u| is an important parameter to control the dynamical behavior in the nonlinear system [20-23].

According to the above analysis, it means that the butterfly attractor reported is a compound structure obtained by merging together two simple attractor after performing a mirror operation.

6. Conclusion

In this Lorenz-like chaotic system, abundant and complex dynamical behaviors have been investigated by nonlinear dynamical method. The forming mechanism of the chaotic attractor is studied, the results show that this new chaotic attractor is a compound structure obtained by merging together two simple attractors after a mirror operation. The chaotic attractors proposed can be also realized with an electronic circuit and have great potential for communication and electronics. The new attractors and their forming mechanism need further to study and explore. And their topological structure and more detailed theory analysis should be completely and thoroughly investigated.

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