

Design and Case Analysis of Advanced High School Mathematics Thinking Activities

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Abstract

The advancement of high school mathematics thinking is an important means and method to cultivate students' own mathematical literacy and improve their ability to solve mathematical problems, and it is also an important part of the current methodology of solving mathematical problems. How to design and organize teaching activities to promote the progressive development of students' mathematical thinking is an important task in mathematics teaching. Teachers need to conduct in-depth analysis of teaching content, find connection points of thinking, accurately grasp teaching difficulties, and construct a logical system of problem chains. This paper explores the development of high school mathematics thinking advancement activities, so as to promote the development of students' thinking and improve their core mathematical literacy.

Keywords

Advanced Mathematics Thinking, Activity Design, Derivative Isomorphism, Model Construction.

1. Introduction

The advancement of high school mathematics thinking is a complex and multi-level process, involving the deepening of mathematical knowledge, the cultivation of abstract thinking, the modeling and solving of practical problems, and other aspects. Mathematics teachers play an irreplaceable leading role in the teaching process, especially a key role in the advancement of students' mathematical thinking. Under the current background of comprehensively implementing core literacy and promoting classroom teaching reform, teachers, based on students' learning situation and knowledge characteristics, organize teaching activities in accordance with the development law of students' mathematical thinking, and construct a model for the advancement of high school mathematics thinking. This helps students build a ladder for the advancement of mathematical thinking, which has important practical value and long-term significance for improving students' core mathematical literacy, especially the literacy of mathematical abstraction, logical reasoning, and mathematical modeling, and promoting the high-quality development of high school mathematics teaching.^[1]

2. Requirements for the Design of High School Mathematics Thinking Advancement Activities

The construction of a high school mathematics thinking advancement model requires students to combine their existing experience and knowledge system, go through a process from intuition to abstraction and from simplicity to complexity. By means of the further development of thinking, they can explain the essence of knowledge, connect with the original thinking structure, and expand the existing system, so as to meet the design requirements of mathematics thinking advancement activities.

2.1. Establish the starting point of thinking advancement and exert the driving effect of problems

The construction of a high school mathematics thinking advancement model first requires defining the objectives of teaching activities for thinking advancement and the requirements for students' mathematical knowledge structure. Based on the key and difficult points of teaching, the starting point of mathematical thinking advancement is determined. On this basis, problem situations are created, with the starting point of mathematical thinking advancement as the origin and problems as the main line, to carry out teaching activities of mathematical thinking and exploration. Therefore, how to select problems carefully and exert the driving effect of mathematical problems naturally becomes the foundation and prerequisite for effectively carrying out teaching activities of mathematical thinking advancement.

2.2. Identify the levels of thinking changes and form an advanced logical chain

Students' thinking transformation does not happen overnight; it goes through a multi-stage developmental process. Based on students' current existing thinking, teachers need to analyze the connotation of knowledge, identify the levels of students' thinking changes, and find the "steps" where students' thinking undergoes transformation. They should consider in what kind of teaching activities students can experience mathematical thinking and cross their current thinking levels, and on this basis, form a logical chain for mathematical thinking advancement. This mainly includes predicting students' difficulties in advancement through analysis of their learning situation, determining the requirements for advanced abilities and the advancement context through textbook analysis, identifying the information integration between knowledge and textbook modules through content analysis, and assessing the degree of information extraction and integration based on existing thinking. All these are conducive to developing and improving students' logical reasoning literacy.

2.3. Build an advancement path and promote the development of thinking variations

Students' thinking changes follow the pattern from simplicity to complexity. The ability to handle simple problems serves as the foundation for complex applications; if even simple problems cannot be dealt with accurately and flexibly, tackling complex ones becomes impossible. Therefore, teachers need to build an advancement path for students from simple mathematical thinking problems to comprehensive ones. The selection of this path depends on the aforementioned "steps" and logical system, enabling thinking variations. Faced with the requirements of progressively deepening mathematical knowledge and advanced development of thinking, teachers should specifically consider the process of students' thinking changes at each stage, design inquiry problems that move from simple to complex and from concrete to abstract, and explore and compare different strategies and methods through various problem variations.^[2] This helps students' thinking develop in a step-by-step and in-depth manner, effectively improving their mathematical abstraction literacy.

2.4. Expand the application space and promote the connection and advancement of thinking

After the first three stages of thinking advancement teaching activities, students have already acquired the knowledge and logical foundation for mathematical thinking advancement. Finally, through flexible application, they carry out thinking integration, expansion and extension. Therefore, teachers need to design comprehensive problems closely related to the content of this section, guide students to integrate the knowledge content of this section, and construct a knowledge system to further expand the application space.^[3] In guiding students to model and apply practical mathematical problems, it can promote the connection and advancement of mathematical thinking and effectively improve their mathematical modeling literacy.

3. Case Analysis of High School Mathematics Thinking Advanced Activity Design

【Case】(2023•New College Entrance Examination Volume I)

Given the function $f(x) = a(e^x + a) - x$.

(1) Discuss the monotonicity of $f(x)$; (2) Prove that when $a > 0$, $f(x) > 2\ln a + \frac{3}{2}$

3.1. Identify the starting point of thinking advancement and give full play to the effect of problem-driven learning

The teaching objective to be achieved in this teaching exploration is to enable students to master the method of constructing isomorphic functions, and use tangent - line scaling for functions, aiming to improve students' mathematical abstraction literacy. The key questions for students to learn are: ① How to carry out isomorphic scaling? ② How to flexibly apply the derivative isomorphic scaling method in different problem situations? The second question is also the difficulty of this lesson.

3.2. Determine the levels of thinking changes and form an advanced logical chain

To achieve the teaching objectives, students' thinking first needs to overcome "How to carry out isomorphic scaling?", and then can reach the thinking progression of "Flexibly applying isomorphic scaling in different situations". We determine the following logical chain of thinking progression: ① Recognize the models of common function isomorphisms: combinations of power functions and exponential - logarithmic functions; ② Prove common scaling inequalities: tangent inequalities of exponential functions and tangent inequalities of logarithmic functions; ③ Master the direct application of simple and common isomorphic scaling models; ④ In different problem situations such as inequality proof, maximum - minimum value solution, and parameter range determination, flexibly use the isomorphic scaling method to solve problems.

3.3. Build an advanced path to promote the development of thinking variants

Step 1: Understand common models of function isomorphism. To explore common isomorphic models of combined functions involving power functions and exponential-logarithmic functions, we first guide students to explore a group of isomorphic combinations of linear functions and exponential-logarithmic functions, enabling them to initially grasp the idea of isomorphism processing. Then, students are asked to independently solve the other two groups of isomorphic cases by analogy, so as to further deepen their understanding of the processing methods. For this purpose, Question 1 and Question 2 are designed as follows, which build a stepping stone for students to learn common isomorphic functions and serve as the first step for the progressive development of students' thinking.

Problem 1: Prove the identities: $xe^x = e^{x+\ln x}$, $x + \ln x = \ln(xe^x)$.

Problem 2: By analogy, prove $\frac{e^x}{x} = e^{x-\ln x}$, $x - \ln x = \ln \frac{e^x}{x}$, $\frac{x}{e^x} = e^{\ln x - x}$, $\ln x - x = \ln \frac{x}{e^x}$

Step 2: Prove commonly - used scaling inequalities. To enable students to recognize and master commonly - used scaling inequalities, it is necessary to lead students to understand and prove the origin of the inequalities from multiple perspectives. For this purpose, design the following Problem 3.

Problem 3: Prove that $e^x \geq x+1$, $\ln x \leq x-1$

Step 3: Master the direct application of simple and common isomorphic scaling models. To enable students to master the derivative - based isomorphic scaling method, lead students to

directly apply it in simple and common isomorphic scaling models and initially experience the derivative - based isomorphic scaling method. For this purpose, design the following Problem 4 and Problem 5.

Problem 4: Prove that $xe^x \geq x + \ln x + 1$, $x + \ln x \leq xe^x - 1$

Problem 5: Can students, by analogy, obtain the inequalities of the remaining expressions of the "brothers of e^x " and the "sisters of $\ln x$ " mentioned earlier?

3.4. Expand the Application Space and Promote the Advanced Development of Thinking Connections

The above process not only reviews relevant knowledge such as exponential - logarithmic operations and tangent lines but also leads students to explore and obtain common isomorphic function and inequality models, providing methods for derivative - based isomorphic scaling. Therefore, when students later deal with problems of inequality proof, finding extreme values, and determining the range of parameters, they will have at least two ideas, namely differentiation and scaling. To further promote the advanced development of students' thinking connections and help them flexibly select and apply this knowledge in practical problem - solving, it is necessary to use comprehensive problems closely related to derivative - based isomorphic scaling, integrate knowledge to form a knowledge system, and then expand the application space of knowledge.

4. Conclusion

Mathematics education should not only focus on knowledge imparting, but also cultivate students' mathematical thinking and problem-solving abilities. How to comprehensively implement core literacy in current high school mathematics teaching is not only an important and urgent theoretical issue, but also a teaching practice problem worthy of in-depth exploration. The ultimate goal of designing high school mathematics thinking advancement activities is to cultivate students' all-round development of mathematical thinking, and to implement and improve their core mathematical literacy. To this end, teachers should strengthen research, accurately grasp the essence of mathematical thinking advancement activities, and carry out precise design. With problems as the main line and thinking advancement as the connotation, teachers should help students deeply understand the nature of mathematics, explore the connections between mathematical knowledge, think about and deal with problems from multiple perspectives, and guide students to establish effective connections between knowledge and cognition. Through the comprehensive application of mathematical knowledge and methods, students' progressive learning of thinking can be promoted, the advancement of mathematical thinking can be continuously driven, and students' core mathematical literacy can be effectively improved, especially the literacy of mathematical abstraction, logical reasoning and mathematical modeling.

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